# The Development of Arithmetic Skills and Concepts from Infancy Through the Early School Years

Before children enter school, and even before they can talk or count, they show signs of early arithmetic abilities. Infants pay attention to amounts of things and changes in quantity. Toddlers notice when there is "more" or "less" of something and can reason about quantities. As their cognitive and verbal skills advance in preschool, children develop an understanding of counting and numbers and begin to solve simple arithmetic problems by manipulating objects. This development provides a foundation for the later skills of addition and subtraction. From kindergarten through the early school years, children deepen their knowledge of addition and subtraction and learn to solve arithmetic problems more quickly and accurately.

Even though young children acquire some arithmetic skills naturally through their interactions with the world around them, they benefit greatly from high-quality math instruction before they enter kindergarten.[1–4] Children who enter kindergarten with strong math skills have higher math and reading achievement at the end of elementary school than children who enter kindergarten with weak math skills.[5] However, high-quality math instruction in preschool does not mean flashcards or rote memorization. Young children learn math best through a combination of play-based, everyday experiences as well as structured, age-appropriate learning activities.[6,7] Beyond preschool, children's arithmetic skills are best supported through instruction that encourages flexible use of problem-solving strategies and that promotes an understanding of arithmetic concepts.

**Young children learn math best through a combination of play-based, everyday experiences as well as structured, age-appropriate learning activities**

### In This Brief

- A summary of research findings on the development of arithmetic skills and concepts from infancy through the early school years
- Practical implications for educators and caregivers of young children from birth through age eight

This brief presents an overview of research on the development of children's arithmetic skills along with related practical implications for educators to support children's early math learning. Although these strategies are geared toward teachers, many can be used by families in home settings as well. The focus is on typically developing children in healthy and supportive caregiving and education settings. However, it is important to keep in mind that individual children show variability in the rate and course of their arithmetic learning. Some of these differences are attributed to varying general cognitive skills, such as executive functions (e.g., the ability to hold information in memory, think flexibly, and regulate behaviors and thinking).[8,9] Language skills also play a role[10–12]: dual language learners, for example, may have the conceptual understanding but may still be developing mathematical vocabulary in both languages.[13–16] Initially, they may be better able to demonstrate their math abilities nonverbally.

**Infants, even in their first week of life, pay attention to number and notice differences in quantity.**

Additionally, the quality of math experiences in children's homes, communities, and child care settings, before entering formal schooling, plays a significant role in their development of early math skills and concepts.[17–21] Research suggests that socioeconomic factors are associated with young children's development of math abilities.[22,23] Given the different factors that influence math learning, some children may exceed the competencies that are described for a particular developmental period, while others may need more time and support to reach that level. A summary of foundations and standards in early arithmetic for infants and toddlers, preschoolers, and early elementary school students in California is presented in Appendix A.

## Sensitivity to Quantity in Infancy

Infants, even in their first week of life,[24,25] pay attention to number and notice differences in quantity.[26–30] For example, infants can tell the difference between two quantities that are significantly different from one another (1:2 ratio), such as a group of 8 dots versus a group of 16 dots.[31] As infants approach their first birthday, they notice even more subtle differences between two quantities (2:3 ratio), such as between groups of 8 dots and 12 dots.[32–34]

Infants also notice when a quantity has increased or decreased. For example, many nine-month-olds have some expectation about what will happen when you add or remove objects from a set.[35] Researchers are able to study infants' expectations about changing quantities by comparing how long they look at events that are unexpected versus expected. Infants react in this way because they find unexpected things more interesting and therefore have a tendency to look longer at them compared to expected things.[35] For example, in one study, researchers showed five-month-olds two dolls, covered the dolls with a screen, and then reached behind the screen to remove one doll (in full view of the infants). When the researchers lowered the screen, they showed either one doll (which would be expected after one doll had been removed from the original two) or two dolls (which would be unexpected). They recorded infants' looking time in both of these scenarios and found that infants looked longer at the unexpected event—when there were two dolls behind the screen.

The finding that infants are sensitive to quantity suggests that humans are born with the capacity to think and learn about numbers.[36] This early, nonverbal sensitivity to quantity lays the foundation for reasoning about changes in quantity in toddlerhood[37] and for arithmetic skills that develop later on.[38–41]

**Infant**
- Notices amounts of things in environment
- Sensitive to changes in quantity

**Toddler**
- Understands more and less
- Knows a few number words but is still learning their exact meaning
- Can add and subtract small approximate quantities

**Preschool**
- Compares two quantities in terms of more or less
- Can add and substract small quantities of concrete objects
- Uses number words for small sets only
- Begins to use counting as a strategy for addition

**Elementary**
- Can add and subtract larger quantities
- Learns arithemtic properties (e.g., 5 + 4 = 4 + 5)
- Uses various strategies to solve problems (e.g., counting all, counting on, decomposition)
- Solves simple arithmetic word problems

# Reasoning About Quantities in Toddlerhood

Toddlers have remarkable skills in reasoning about quantities. Researchers have gained insight into toddlers' emerging arithmetic ability by presenting problems nonverbally—as a set of objects that is transformed by adding or removing items—and by involving only very small quantities of objects (up to three total). This line of work has revealed that toddlers understand that addition results in having more of something and that subtraction results in having less of something.[42,43] In fact, when asked to add one set of objects to another, toddlers can even figure out about how many of those objects there would be in total.[42] That is, toddlers are capable of doing approximate arithmetic.

What does approximate arithmetic look like? Imagine you showed a toddler two strawberries before placing them into a covered basket and then showed one more strawberry before dropping it into the basket. Few two-and-a-half-year-olds would be able to figure out that there are now exactly three strawberries in the basket. However, most two-and-a-half-year-olds—and even many one-and-a-half-year-olds[44]—would show an understanding that there are more strawberries in the basket than there were before. Even more striking, if you asked two-and-a-half-year-olds to give you the same number of strawberries you had put into the basket, many would give you a quantity that is pretty close to three,[42] although they would probably not be able to reliably give you exactly three. Similarly, if you asked them to pick the strawberries out of the basket one at a time, they might stop looking in the basket once they had pulled out three strawberries.[43]

Even though children of this age can solve simple nonverbal arithmetic problems in an approximate way, they become more attentive to the exact number of objects in a set. Some research has found that toddlers can think about quantities with more precision depending on characteristics of objects within the set.[45] For example, if you put one strawberry and one blueberry on your plate and asked a two-and-a-half-year-old to make her plate look like yours, she would most likely put exactly one strawberry and one blueberry on her plate. The argument is that when the objects within a small set are different from each other, toddlers may spontaneously pay more attention to

### Practical Implications for Adults Working with Infants and Toddlers

A key way to support infants' and toddlers' early quantitative ability is to draw their attention to number and quantity in the world around them. This approach can be taken through play with sets of objects that can be grouped, compared, combined, and separated. Using number words and quantitative language (e.g., "more," "less," "equal") during interactions with children, even before they can talk (or if it seems like they do not understand), lays the foundation for building children's mathematical vocabulary in the coming years. This vocabulary will help them develop arithmetic skills and conceptual understanding during preschool and early elementary school.[10,46–48]

Research-Based Strategies:

- Refer to concrete objects when counting rather than only counting by rote.[48] For example, when counting to three, count three blocks or three shovels in the sandbox.
- During play and daily routines, offer opportunities for grouping and combining sets of objects. When doing so, count the objects and label the total set size.[49] For example, add a car to a set of two and count "1, 2, 3," and then say, "we have three cars altogether."
- Use words to compare quantities of objects, such as who has "more" or "less."[46]

the exact quantity of the set. Another factor that may affect children's attention to exact quantities is their understanding of the meaning of number words, such as "two" or "three."[45] These emerging skills in thinking about the exact number of objects in a set is a precursor to developing nonverbal arithmetic skills during the preschool years, when they begin to add and subtract small numbers of objects.[42]

## Arithmetic in the Preschool Years

During the preschool years, many children have moved beyond reasoning about quantities in an approximate way and start to think about transforming quantities of objects with more precision.[42,50,51] For instance (using the strawberry basket example given above), if you put two strawberries into a basket, add one more to the basket, and then ask a four-year-old child to give you the same number of strawberries as there are in your basket, he would most likely be able to give you exactly three strawberries. However, children of this age are still building their vocabulary of quantity words (e.g., "more," "less") and arithmetic operations (e.g., "add," "subtract," "plus," "minus")[52] as well as the ability to keep track of information in their mind.[53,54]

Because of these developmental characteristics, it is easier to see preschoolers' emerging ability to add and subtract when we present them with problems nonverbally—using small quantities of objects (e.g., strawberries, blocks, vehicles)—than when we present them with problems verbally, such as "How much is two plus one?"[42,50,51]

**Around age four, children use counting and number words more accurately to label quantities of objects and show emerging skills in verbal arithmetic.**

As children's skills in nonverbal arithmetic improve, they become comfortable with reasoning about increasingly larger quantities.[42,50] For example, from ages two and a half to four, they are most comfortable adding and subtracting within a total set of about two to three concrete objects.[42] However, starting at around age four, they become more comfortable adding or subtracting within a total set of about five to six concrete objects.[42,50]

### Practical Implications for Adults Working with Preschoolers

Children's arithmetic skills during preschool can be supported by inviting them to engage in tasks that involve quantitative reasoning, such as counting, comparing, combining, and separating sets of objects. During these activities, reinforce the use of number words and mathematical language (e.g., "more," "less," "add," "take away," "all," "some," "none"). At this age, children are ready to reason quantitatively with sets of five objects or larger based on their current skill level. As children engage in problem solving using concrete objects, they develop a deeper understanding of addition and subtraction.

Research-Based Strategies:

- Support understanding of one-to-one correspondence through daily interactions and play, such as setting out four plates on the table for four people to eat.
- Compare two groups of objects and talk about which group has more.[56] Start by comparing quantities that are very different (e.g., 10 vs. 3) and gradually work toward comparing quantities that are similar (e.g., 6 vs. 7).[57]
- Support children's understanding of the numerical value of number words (e.g., that the word "five" means exactly five objects). For example, encourage children to create sets of an exact number[42] through play and daily routines, such as trying to find five teddy bears hidden in a room or putting three orange slices on a plate.
- Invite children to add or take away one object to a set and find the total. For example, ask, "We had five teddy bears and we found one more. How many do we have now?" Taking this approach repeatedly will reinforce the connection between the count sequence and the concept of addition (e.g., that adding one more object to a set results in having a set size that is one number higher in the count list[58,59]). Gradually, you can work on adding two more or taking away two.
- Invite children to make predictions about how many objects will be left after you take some objects away or how many there will be after you add some objects.[55] For example, during mealtime, have a child count six strawberries and put them on his plate, then you put one more strawberry onto the child's plate. See if he can predict how many strawberries are on the plate now, then invite him to count and check his prediction.

Around age four, children use counting and number words more accurately to label quantities of objects and start to show emerging skills in verbal arithmetic.[50] One of the early signs of this bridge from nonverbal to verbal arithmetic can be seen in children's ability to make a verbal prediction about what a set size would be after items have been added or removed. For example, imagine that you showed a preschooler a set of seven tangerines and asked her to count them to figure out how many there were. Then you cover the collection of tangerines and add two more, saying "now we have two more tangerines. How many tangerines do we have altogether?" In this scenario, children's verbal prediction about the total number of tangerines is either exact or close to the correct number.[55] Children of this age also gain skill in solving verbally presented arithmetic problems (e.g., "How much is 3 + 1?) and tend to be most comfortable solving problems involving numbers between one and six.[50]

## Arithmetic Development in Early Elementary Grades: Concepts and Strategies

From kindergarten through the early school years, children are on the road to mastering conventional, verbal addition and subtraction (e.g., 9 + 6 = ___ ) . During this time, they gain an understanding of arithmetic concepts and learn to use different strategies to solve arithmetic problems with efficiency and accuracy. They also become more advanced in their understanding of word problems and how to solve them.

### Developing Understanding of Arithmetic Concepts

Conceptual knowledge of arithmetic provides a foundation for solving arithmetic problems with understanding. Conceptual arithmetic knowledge includes understanding properties of arithmetic, such as knowing that 5 + 4 = 4 + 5 (addition property of commutativity) and that 5 + 4 – 4 = 5 (inverse property of subtraction). Other conceptual knowledge includes understanding that the counting system is a pattern that repeats every 10 numbers (i.e., the base-10 structure of the number system). This understanding helps children approach problems more flexibly rather than apply procedures by rote. In turn, this flexibility helps them be more efficient and avoid mistakes.[60–63]

By kindergarten age, many children tend to show a basic understanding of the commutativity property when solving problems with physical objects.[51,64–66] For example, children of this age understand that, if you first give them three orange blocks and then give them two red blocks, they will have the same total number of blocks as if you had given them the two red blocks first and the three orange blocks second. However, children may not be able to apply this basic knowledge to verbally presented problems (i.e., without physical objects) until early elementary school.[65,67,68] For example, if you presented the written problems 6 + 3 and 3 + 6 side by side and asked a six-year-old to tell you whether those two problems have the same answer, she might not consistently say "yes." Similarly, an understanding of the inverse relation between addition and subtraction on verbally presented problems (e.g., "Can you use your answer to 9 + 4 to help you figure out the answer to 13 – 4?") seems to emerge early in elementary school.[69]

Children's understanding of the base-10 number system also develops substantially during this period. Early on, before children have been exposed to formal math instruction, they tend to think about quantities as collections of units rather than as groups of 10s and 1s.[70] For example, if you asked a five-year-old to show you 13 unit blocks (with cubes that represent 1s and bars that represent 10s), he would probably count out 13 individual unit blocks. However, from kindergarten through the early school years, children increasingly represent numbers greater than 10 using a combination of 10s and 1s. For example, if asked to show 13 using unit blocks, a seven-year-old would likely set out one bar (representing 10) along with three unit cubes.[71–73]

## Development of Arithmetic Strategies

The strategies children use to add and subtract are an important part of their arithmetic development. Learning to use efficient mental arithmetic strategies usually leads to more accurate responses.[74–76] Successful use of these strategies requires instructional guidance by teachers and repeated opportunities to apply them in meaningful ways. From kindergarten through second grade, children go through a major transformation in the way they approach arithmetic problems. Early on, children tend to use strategies that are inefficient and result in more errors, such as counting on five fingers and three more fingers to add five plus three but losing track while counting. Later on, they begin to use more efficient and accurate strategies, such as thinking, "I know 5 + 2 = 7, and three is one more than two, so 5 + 3 = 8."[77,78] The strategy they choose on a particular problem tends to be the most efficient they feel they can use without making mistakes[79]. For example, a child will count using fingers to solve an addition problem if she feels it will be more likely to get her to the correct answer than counting verbally, but, if she feels confident in solving the problem without using fingers, she will choose a verbal counting strategy because it is more efficient. However, as described below, children generally move toward choosing more advanced strategies over time.[79]


**Counting strategies.** Kindergartners make use of their counting skills to do addition and subtraction, and they tend to use concrete objects to help them keep track when counting. For example, when given the problem 6 + 3, a typical kindergartner's go-to strategy would be to count out six objects, such as teddy bear counters, and then count out three more before recounting the whole set of nine.[77] This "count-all" strategy supports children's earliest verbal arithmetic ability.

**Kindergartners make use of their counting skills to do addition and subtraction, and they tend to use concrete objects to help them keep track when counting.**

By the time they are in first grade, children become more efficient when using counting strategies: they begin counting up from one of the numbers in an addition problem instead of counting from one.[77] For example, when asked, "How much is 5 + 3?", a child might say aloud or whisper to himself "5" and then count on three fingers "6, 7, 8." This strategy can be challenging for children because it requires them to be able to start counting from numbers other than one. It takes time and a lot of practice for children to comfortably use "counting-on" strategies. Different types of counting-on strategies reflect different levels of conceptual understanding. Early on, before children understand the commutative property of addition, children typically count on from the number that is presented first.[77] For example, for the problem 3 + 6, they will start at three and count up six. However, children who understand the commutative property of addition might choose instead to start at the larger number (in this case, six) and count up the smaller number (three) because it is more efficient.

**Memory-based strategies.** As children master counting strategies, they begin to memorize number facts.[77] For example, after solving the problem 5 + 3 using a counting strategy many times, they will eventually remember that the answer to 5 + 3 is 8 and will be less likely to rely on counting to solve that problem in the future. At first, children tend to memorize sums and differences of two single-digit numbers (e.g., 6 + 3 = 9, 7 – 2 = 5), 10s (e.g., 10 + 10 = 20, 30 – 10 = 20), or multiples of 10s (e.g., 200 + 300 = 500, 400 – 100 = 300). When they are given a problem they have not yet memorized, many children rely on the number facts they do know to help them solve problems in their head. For example, they might know that 5 + 5 = 10, so, when asked to solve 5 + 7, they might say, "I know that 5 + 5 = 10, and 7 is 2 more than 5, so it's 12." This type of strategy involves decomposing numbers, such as separating seven into five and two, and reflects an understanding of part-whole relationships between numbers.[60,61] It is the most efficient way for children to solve a problem in their head (without the use of counting tools or paper) and generally leads to more accurate responses than counting strategies do.[75,76,80–82]

## Development of Arithmetic Word Problem Solving

Learning to solve word problems represents a major achievement for elementary school students. Word problem solving requires a high level of reasoning that includes conceptual arithmetic knowledge and fluency with problem-solving procedures.[83,84] Early on, children typically solve verbally presented story problems by counting objects or fingers.[85] For example, many older kindergartners (approaching age six) can solve a simple story problem, such as the following, with the help of concrete objects or fingers: "Sam has four crayons. He got two more crayons. How many crayons does he have now?"[50] Slightly older students (e.g., first graders) can even solve problems with a more complex structure, such as this: "Sam has three crayons. He got some more. Now he has seven crayons. How many crayons did he get?" They might solve this problem using a very intuitive strategy, such as by counting out three objects, adding objects to the set one by one while counting aloud until they get to seven, and finally counting the number of objects they added to get the answer of four.[86]

In the early grades, children learn to solve problems involving larger numbers. However, without concrete objects as a tool, solving more advanced word problems may be a challenge for students. Another challenge is that they are exposed to a greater variety of word problems, such as those involving comparisons or starting with an unknown number (e.g., Sam had some crayons. He got 9 more. Now he has 21. How many crayons did he have in the beginning?). It is often not intuitive from the language of word problems whether to use addition, subtraction, or a combination to solve the problem. With instructional support, students learn to read the problem completely and represent the problem schematically to help them identify the problem structure.[87–89] Following these first steps that emphasize comprehension, they learn to write a problem sentence before completing calculations needed to solve the problem.[88,90,91]

**By the time they are in first grade, children become more efficient when using counting strategies; they begin counting up from one of the numbers in an addition problem instead of counting from one.**

### Practical Implications for Adults Working with Early Elementary Students

As children develop skill in verbal arithmetic, how they solve addition and subtraction problems is just as important as solving problems correctly. Introducing children to different problem-solving strategies and encouraging children to become flexible in their use of different strategies will increase their fluency, build their knowledge of simple number facts, and ultimately help them solve arithmetic problems with greater accuracy.[7,74,77] Flexible problem solving is supported by a conceptual understanding of arithmetic[61] and by teaching children how to use number facts they know to help them solve problems they don't know the answer to.[92,93]

#### Research-Based Strategies:

- When using concrete objects to support counting strategies, choose objects that will build children's conceptual understanding. For example, use unit blocks or interlocking counting cubes, which highlight the base-10 structure of the number system.[62,63] Phase out the use of concrete objects as children gain fluency in implementing counting strategies.
- The use of number grids or number lines as tools for arithmetic problem solving can help children progress from using concrete objects toward using verbal and mental counting strategies. For example, children can use a number line to support a "counting-on" strategy when adding 7 + 5 (e.g., start at 7 and count on to 12 while pointing to each space on the number line). Similarly, 0–10 or 0–100 number grids can be used as board games (e.g., Chutes and Ladders) and provide opportunities to practice counting-on strategies to move tokens. For example, if a child's token is on four and she rolled a three on a die, she could start at "4" and count on three fingers, "5, 6, 7."[94,95]

- Repeated practice of number facts (e.g., memorizing doubles, 10-combinations) can provide a good foundation for solving problems that are not memorized.[92,93,96] For example, if a child has memorized $6 + 6$, encourage him to use that number fact to solve a near-double problem, such as $6 + 7$. In the same vein, children can be encouraged to use their knowledge of 10-combinations, such as $7 + 3 = 10$, to solve problems that cross over into the next set of 10, such as $7 + 6$ (e.g., $7 + 6 = 7 + 3 + 3 = 10 + 3 = 13$).
- Draw connections between addition and subtraction problems to highlight the inverse relationship between these two operations (e.g., show how the problem $5 + 3$ can help them figure out the answer to the problem $8 – 3$)[68,69] rather than work on addition and subtraction problems separately.
- Emphasize full comprehension of word problems rather than pick key words out of context, and help children identify the problem structure before choosing a calculation strategy.[88,90,97–99]
- For word problems, draw connections between problems with similar structures (e.g., a word problem about balloons expressed as $? + 5 = 12$ is similar to a word problem about cars expressed as $? + 9 = 14$). This strategy will facilitate children's conceptual understanding of problems that may appear different on the surface but actually have similar arithmetic structures.[99]

## Conclusion

As we have described in this brief, children's arithmetic development begins in the first weeks of life, when they show an innate sensitivity to quantities.[24,25] It progresses in the toddler and preschool years to reasoning about small quantities of concrete objects.[42,50] Finally, students in early elementary school develop more conceptual understanding, the ability to follow math procedures, and higher-level problem-solving skills.[100] Caregivers and teachers play an important role in supporting this trajectory. When young children's natural capacity to reason about quantities is supported, they acquire the vocabulary, strategies, and knowledge needed to do conventional arithmetic fluently and accurately.

**<<< Possibly, at very last stage, add list of other briefs >>>**

# Appendix A: California Early Learning Foundations and Standards in Arithmetic

## California Infant-Toddler Learning Foundations

**Foundation:** Number Sense

The developing understanding of number and quantity

| 8 months | 18 months | 36 months |
|---|---|---|
| At around 8 months of age, children usually focus on one object or person at a time, yet they may at times, hold two objects, one in each hand. | At around 18 months of age, children demonstrate understanding that there are different amounts of things. | At around 36 months of age, children show some understanding that numbers represent how many and demonstrate understanding of words that identify how much. |

Source: California Infant-Toddler Learning Foundations

## Preschool Learning Foundations in Mathematics

**Number Sense:** Substrand 2.0

| | At around 48 months of age | At around 60 months of age |
|---|---|---|
| Number Sense | **2.0 Children begin to understand number relationships and operations in their everyday environment.**<br>2.1 Compare visually (with or without counting) two groups of objects that are obviously equal or nonequal and communicate, "more" or "same."<br>2.2 Understand that adding to (or taking away) one or more objects from a group will increase (or decrease) the number of objects in the group.<br>2.3 Understand that putting two groups of objects together will make a bigger group.<br>2.4 Solve simple addition and subtraction problems, nonverbally (and often verbally) with a very small number of objects (sums up to 4 or 5). | **2.0 Children expand their understanding of number relationships and operations in their everyday environment.**<br>2.1 Compare by counting or matching two groups of up to five objects and communicate, "more," "same as," or "fewer" (or "less").<br>2.2 Understand that adding one or taking away one changes the number in a small group of objects by exactly one.<br>2.3 Understand that putting two groups of objects together will make a bigger group and that a group of objects can be taken apart into smaller groups.<br>2.4 Solve simple addition and subtraction problems with a small number of objects (sums up to 10), usually by counting. |
| | | |
| Mathematical Reasoning | **1.0 Children use mathematical thinking to solve problems that arise in their everyday environment.**<br>1.1 Begin to apply simple mathematical strategies to solve problems in their environment. | **1.0 Children expand the use of mathematical thinking to solve problems that arise in their everyday environment.**<br>1.1 Identify and apply a variety of mathematical strategies to solve problems in their environment. |

Source: California Preschool Learning Foundations in Mathematics

## Common Core State Standards for Mathematics: Kindergarten–Grade 2

**Domains:** Operations and Algebraic Thinking; Number and Operations

| | Kindergarten | Grade 1 | Grade 2 |
|---|---|---|---|
| Operations and Algebraic Thinking | K.OA | 1.OA | 2.OA |
| | **Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.** | **Represent and solve problems involving addition and subtraction.** | **Represent and solve problems involving addition and subtraction.** |
| | 1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.<br>2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.<br>3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).<br>4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.<br>5. Fluently add and subtract within 5. | 1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.<br>2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. | 1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. |
| | | **Understand and apply properties of operations and the relationship between addition and subtraction.** | **Add and subtract within 20.** |
| | | 3. Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)<br>4. Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8. | 2. Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. |

| | Kindergarten | Grade 1 | Grade 2 |
|---|---|---|---|
| Operations and Algebraic Thinking | | 1.OA | 2.OA |
| | | **Add and subtract within 20.** | **Work with equal groups of objects to gain foundations for multiplication.** |
| | | 5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).<br>6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). | 3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.<br>4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. |
| | | **Work with addition and subtraction equations.** | |
| | | 7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.<br>8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = • – 3, 6 + 6 = •. | |

| | Kindergarten | Grade 1 | Grade 2 |
|---|---|---|---|
| Number and Operations | K.NBT | 1.NBT | 2.NBT |
| | **Work with numbers 11–19 to gain foundations for place value.** | **Extend the counting sequence.** | **Understand place value.** |
| | 1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. | 1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. | 1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:<br>a. 100 can be thought of as a bundle of ten tens—called a "hundred."<br>b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).<br>2. Count within 1000; skip-count by 2s, 5s, 10s, and 100s. CA<br>3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.<br>4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. |

| | Kindergarten | Grade 1 | Grade 2 |
|---|---|---|---|
| Number and Operations | | 1.NBT | 2.NBT |
| | | **Understand place value.** | **Use place value understanding and properties of operations to add and subtract.** |
| | | 2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:<br>a. 10 can be thought of as a bundle of ten ones—called a "ten."<br>b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.<br>c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).<br>3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. | 5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.<br>6. Add up to four two-digit numbers using strategies based on place value and properties of operations.<br>7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.<br>7.1 Use estimation strategies to make reasonable estimates in problem solving. CA<br>8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.<br>9. Explain why addition and subtraction strategies work, using place value and the properties of operations. |

| | Kindergarten | Grade 1 | Grade 2 |
|---|---|---|---|
| Number and Operations | | 1.NBT<br><br>**Use place value understanding and properties of operations to add and subtract**<br><br>4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relation-ship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.<br><br>5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.<br><br>6. Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. | |

Source: Common Core State Standards for Mathematics

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