

# Introduction to the California Mathematics Framework

Ellen Barger



**CISC**  
Curricular and Improvement  
Support Committee



**Introduction  
to the  
California  
Mathematics  
Framework**

# **Ellen Barger**

**Associate Superintendent, C&I  
Santa Barbara County Education Office**

**Chair: Curricular and Improvement Support  
Committee of the CA County Superintendents**



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Support Committee

# California Mathematics Framework: **In This Session**



**1. Orient to the structure, features and organizing concepts**

**2. Provide some entry points for use & implementation**

Description: Photo of Ellen Barger as a pre-schooler

A society without **mathematical affection** is like a city without concerts, parks, or museums. To miss out on mathematics is to live without an opportunity to play with beautiful ideas and see the world in a new light.

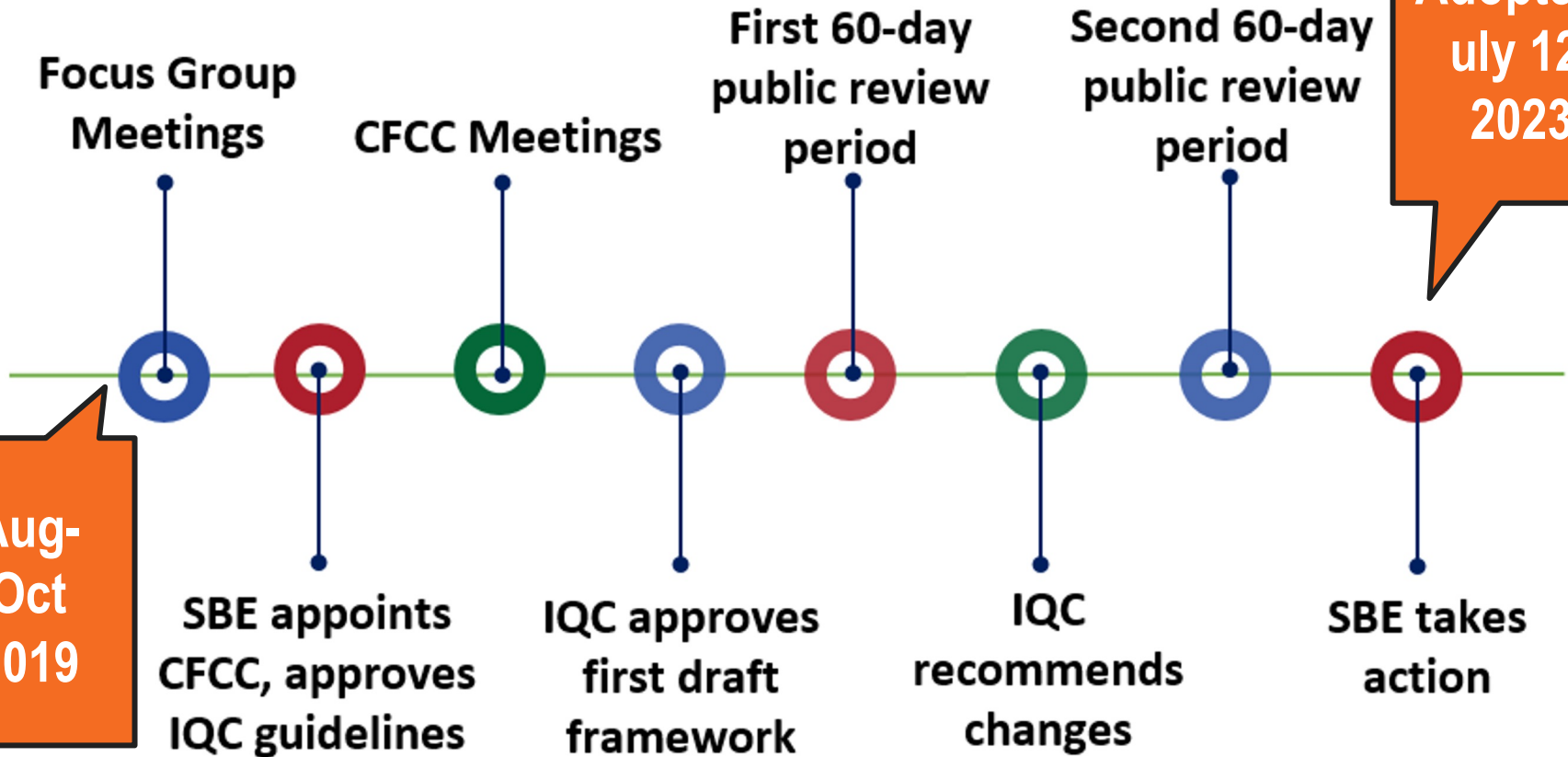
—Francis Su (2020)

# 1. Orient to the structure, features and organizing concepts



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# Framework Development Process



California Mathematics Framework Summary
Chapter 1: Mathematics for All: Purpose, Understanding, and Connection
Chapter 2: Teaching for Equity and Engagement
Chapter 3: Number Sense
Chapter 4: Exploring, Discovering, and Reasoning With and About Mathematics
Chapter 5: Mathematical Foundations of Data Science
Chapter 6: Mathematics: Investigating and Connecting, Transitional Kindergarten through Grade 5
Chapter 7: Mathematics: Investigating and Connecting, Grades 6 through 8
Chapter 8: Mathematics: Investigating and Connecting, High School
Chapter 9: Structuring School Experiences for Equity and Engagement
Chapter 10: Supporting Educators in Offering Equitable and Engaging Mathematics Instruction
Chapter 11: Technology and Distance Learning in the Teaching of Mathematics
Chapter 12: Mathematics Assessment in the 21st Century
Chapter 13: Instructional Materials to Support Equitable and Engaging Learning of the California Common Core State Standards for Mathematics
Chapter 14: Glossary: Acronyms and Terms
Appendix A: Mathematical Progressions within the High School Pathways and Key Mathematical Ideas to Promote Student Success in Introductory University Courses in Quantitative Fields
Appendix B: Works Cited
Appendix C: Vignettes

*How is The  
Mathematics  
Framework  
Structured?*



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Chapter 1: Mathematics for All: Purpose, Understanding, and Connection

Chapter 2: Teaching for Equity and Engagement

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Chapter 14: Glossary: Acronyms and Terms

Appendix A: Mathematical Progressions within the High School Pathways and Key Mathematical Ideas to Promote Student Success in Introductory University Courses in Quantitative Fields

Appendix B: Works Cited

Appendix C: Vignettes

## Introduction

## 3 Progressions - across TK-12

## Grade Band Chapters

## In-Depth Supports, Systems, and Structures

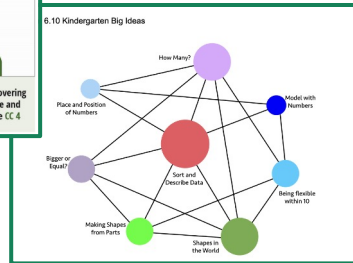
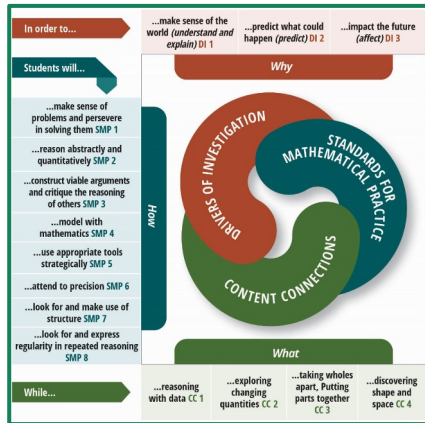
## Resources... including ALL Vignettes



Chapter 1: Mathematics for All: Purpose, Understanding, and Connection

Chapter 2: Teaching for Equity and Engagement

## Chapter 1



## Chapter 2

3 Dimensions of Systemic Change

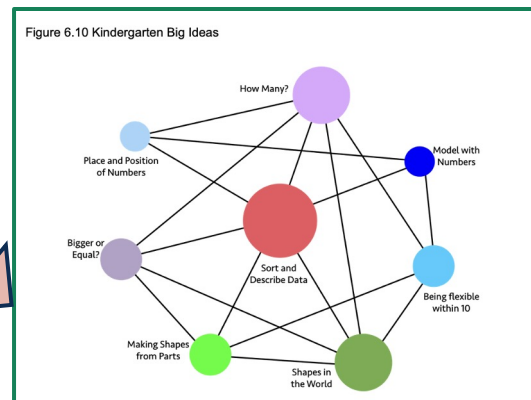
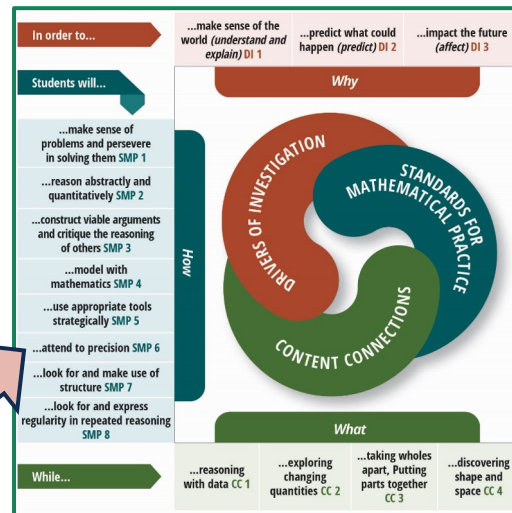
5 Components of Equitable and Engaging Teaching

# Chapter 1

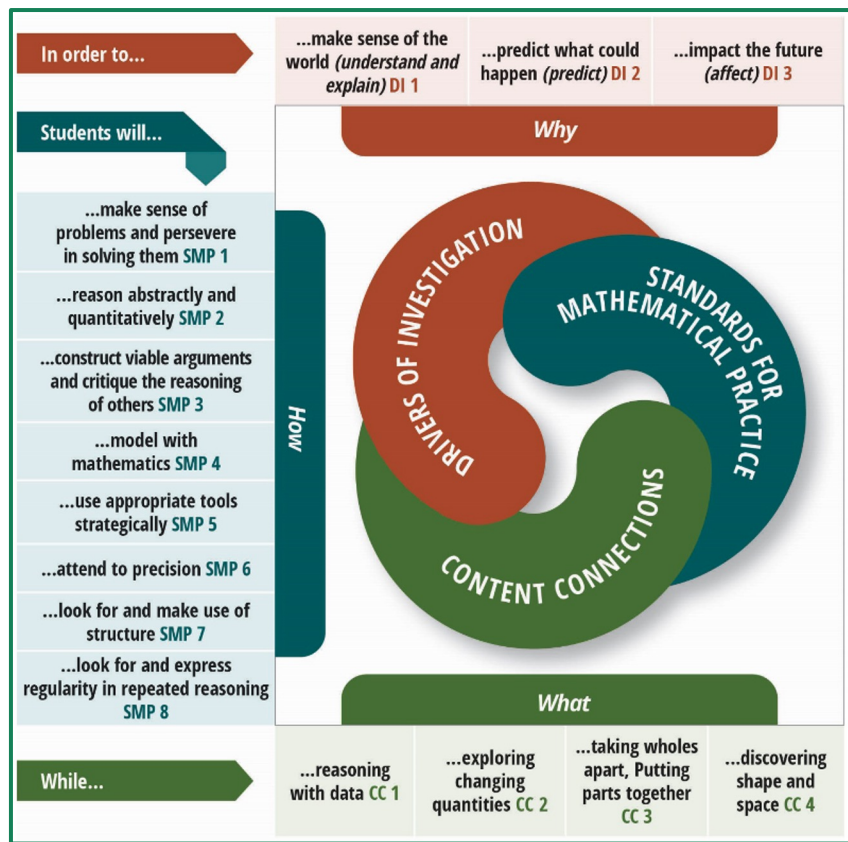
Line Numbers (in the draft form)

8	Mathematics Framework Chapter 1: Mathematics for All: Purpose, Understanding, and Connection .....	1
10	Introduction .....	2
11	Audience .....	3
12	Why Learn Mathematics? .....	5
13	What We Know about How Students Learn Mathematics .....	5
14	Mathematics as Launchpad or Gatekeeper: How to Ensure Equity .....	5
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**WHY** Learn Math  
**HOW** Students Learn  
**EQUITY**



# Chapter 1



## WHY

*Drivers of Investigation (DI)*

## HOW

*Standards for Mathematical Practice (SMPs)*

## WHAT

*Content Connections (CC)*

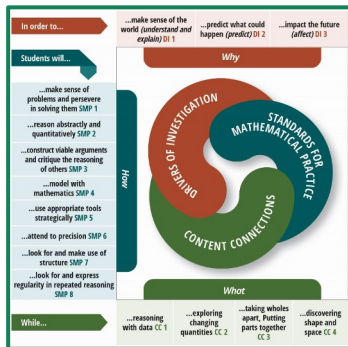
# Chapter 1

...make sense of the world (*understand and explain*) DI 1

...predict what could happen (*predict*) DI 2

...impact the future (*affect*) DI 3

*Why*



**Drivers of Investigation: elicit curiosity and motivate students to engage deeply with authentic mathematics.** They aim to provide a reason to care about mathematical work.

## Students will...

...make sense of problems and persevere in solving them **SMP 1**

...reason abstractly and quantitatively **SMP 2**

...construct viable arguments and critique the reasoning of others **SMP 3**

...model with mathematics **SMP 4**

...use appropriate tools strategically **SMP 5**

...attend to precision **SMP 6**

...look for and make use of structure **SMP 7**

...look for and express regularity in repeated reasoning **SMP 8**

How

## Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and make use of repeated reasoning

## Students will...

...make sense of problems and persevere in solving them **SMP 1**

...reason abstractly and quantitatively **SMP 2**

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...look for and express regularity in repeated reasoning **SMP 8**

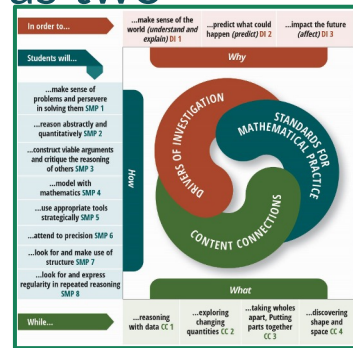
How

## Practice Standards (SMPs):

The SMPs embed the habits of mind and habits of interaction that form the basis of math learning.

To teach mathematics for understanding, it is essential to **actively and intentionally** cultivate students' use of the SMPs.

SMPs must be **taught as carefully and practiced as intentionally** as the content standards, as two halves of a powerful whole



# Chapter 1

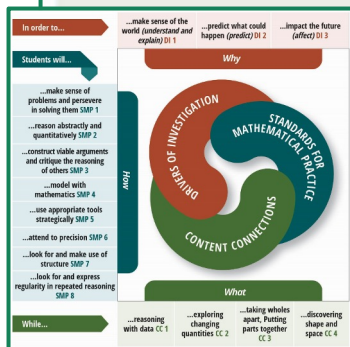
*What*

...reasoning  
with data **CC 1**

...exploring  
changing  
quantities **CC 2**

...taking wholes  
apart, Putting  
parts together  
**CC 3**

...discovering  
shape and  
space **CC 4**

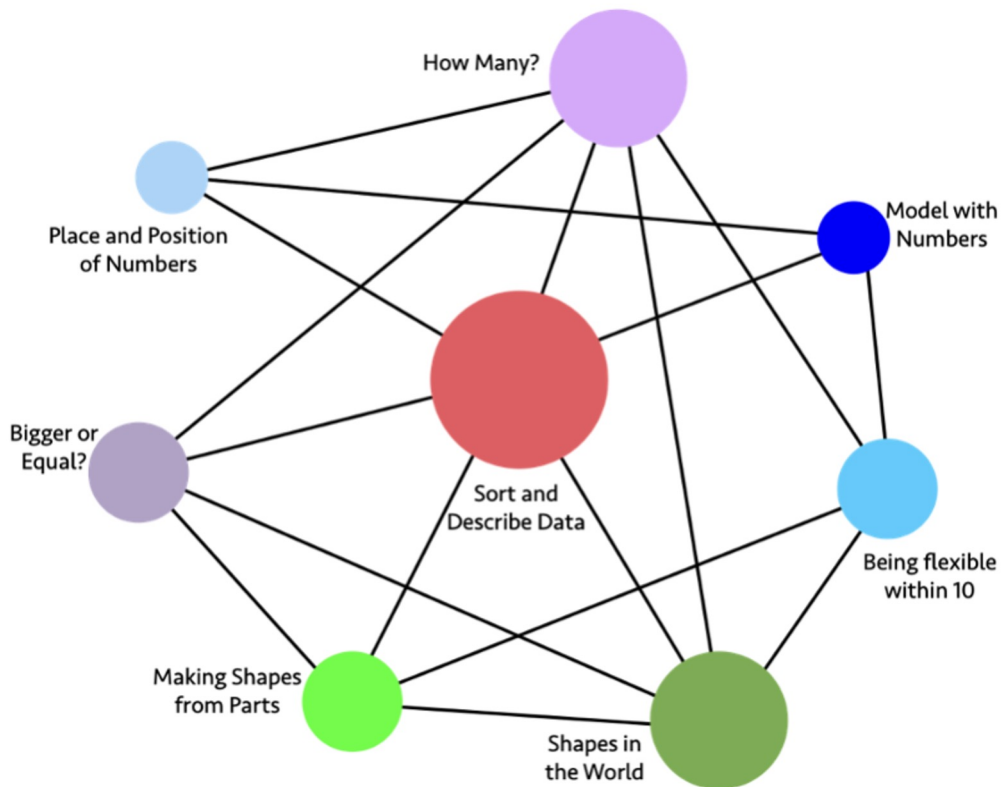


The Content Connections: The four organizing categories of all TK-12 Math Content.

The understandings, skills, and dispositions of all content students learn in school mathematics

# Chapter 1

*Planning Around  
and Teaching to*  
**BIG  
IDEAS**





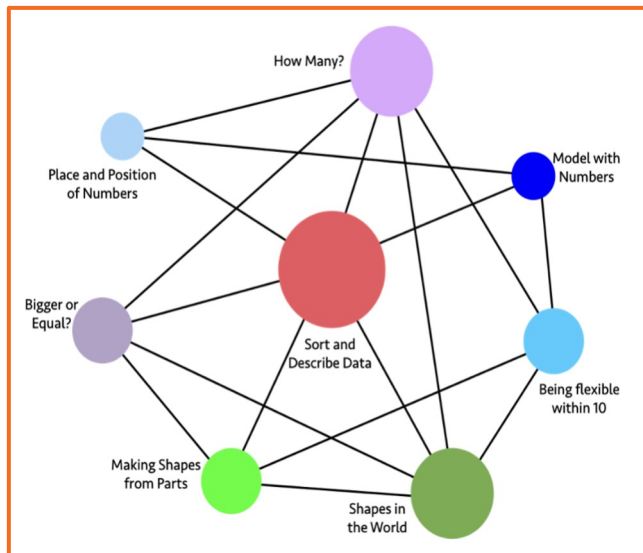
# From Content Connections to Big Ideas

...reasoning with  
data CC1

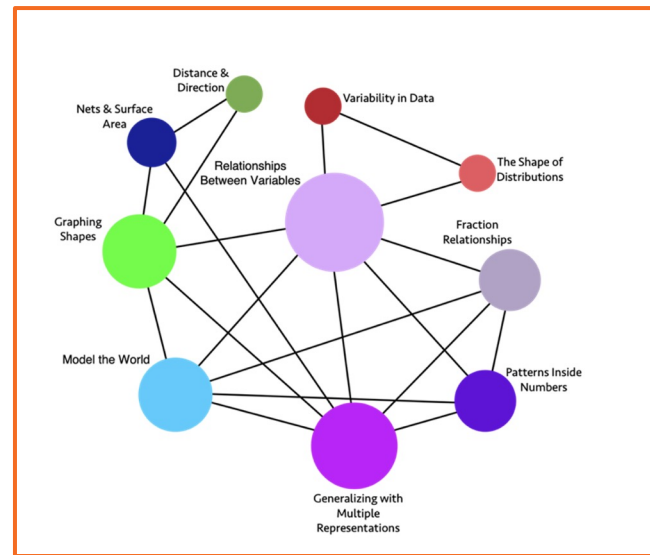
...exploring  
changing  
quantities CC2

...taking wholes  
apart...putting  
parts together CC3

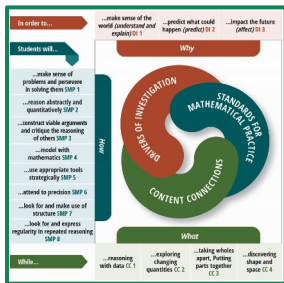
...discovering  
shape and space  
CC4



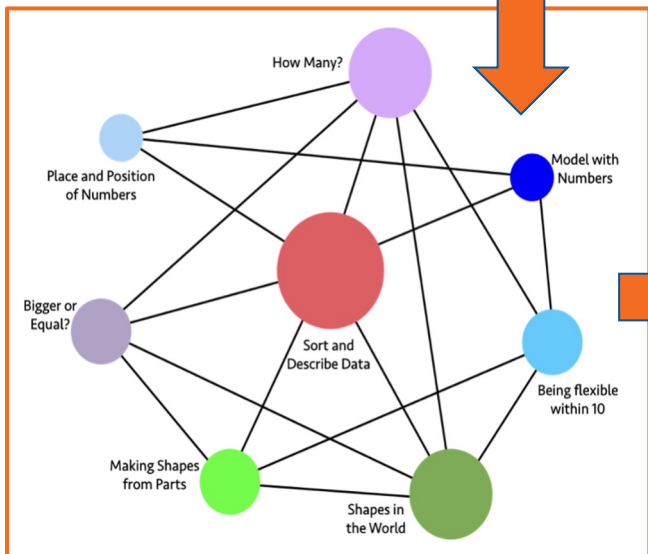
Kindergarten Big Ideas



6th Grade Big Ideas



- ...reasoning with data CC1
- ...exploring changing quantities CC2
- ...taking wholes apart...putting parts together CC3
- ...discovering shape and space CC4



Content Connections	Big Ideas	Kindergarten Content Standards
Exploring Changing Quantities	<b>Bigger or Equal?</b>	<b>CC.4, CC.5, CC.6, MD.2, G.4:</b> Identify a number of objects as greater than, less than, or equal to the number of objects in another group. Justify or prove your findings with number sentences and other representations.
Taking Wholes Apart, Putting Parts Together	<b>Being Flexible within 10</b>	<b>OA.1, OA.2, OA.3, OA.4, OA.5, CC.6, G.6:</b> Make 10, add and subtract within 10, compose and decompose within 10 (find two numbers to make 10). Fingers are important.
Taking Wholes Apart, Putting Parts Together	<b>Place and position of numbers</b>	<b>CC.3, CC.5, NBT.1:</b> Get to know numbers between 11 and 19 by name and expanded notation to become familiar with place value, for example: $14 = 10 + 4$ .
Taking Wholes Apart, Putting Parts Together	<b>Model with numbers</b>	<b>OA.1, OA.2, OA.5, NBT.1, MD.2:</b> Add, subtract, and model abstract problems with fingers, other manipulatives, sounds, movement, words, and models.
Discovering Shape and Space	<b>Shapes in the World</b>	<b>G.1, G.2, G.3, G.4, G.5, G.6, MD.1, MD.2, MD.3:</b> Describe the physical world using shapes. Create 2-D and 3-D shapes, and analyze and compare them.
Discovering Shape and Space	<b>Making shapes from parts</b>	<b>MD.1, MD.2, G.4, G.5, G.6:</b> Compose larger shapes by combining known shapes. Explore similarities and differences of shapes using numbers and measurements.

“Students benefit from **viewing mathematics as** a vibrant, **interconnected,** beautiful, relevant, and creative **set of ideas.**”

*The 2023 California Mathematics Framework, Chapter 2, page 9, lines 170-171*

AND TEACHERS DO  
TOO!!

# CH. 2: TEACHING for EQUITY and ENGAGEMENT

## Chapter 2

### 3 Dimensions of Systemic Change

5 Components of  
Equitable and  
Engaging Teaching

- An **assets-based approach** to instruction
- Active engagement through **investigation and connection**
- Cultural and personal **relevance**

# CH. 2: TEACHING for EQUITY and ENGAGEMENT

## Chapter 2

3 Dimensions of Systemic Change

5 Components of Equitable and Engaging Teaching

### FIVE COMPONENTS OF EQUITABLE AND ENGAGING TEACHING FOR ALL STUDENTS



COMPONENT 1:  
PLAN TEACHING AROUND BIG IDEAS



COMPONENT 2:  
USE OPEN, ENGAGING TASKS



COMPONENT 3:  
TEACH TOWARD SOCIAL JUSTICE



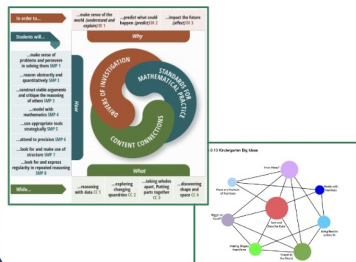
COMPONENT 4:  
INVITE STUDENT QUESTIONS AND CONJECTURES



COMPONENT 5:  
PRIORITIZE REASONING AND JUSTIFICATION

*excerpted from the 2023 California Mathematics Framework*

## Chapter 1



## Chapter 2

3 Dimensions of Systemic Change

5 Components of Equitable and Engaging Teaching

Chapter 3: Number Sense

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Chapter 7: Mathematics: Investigating and Connecting, Grades 6 through 8

Chapter 8: Mathematics: Investigating and Connecting, High School

# CH. 3: Number Sense

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**Developmental  
Progression Description**

**FLUENCY**

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# CH. 3: Number Sense

**Pages 9-25: Descriptions, examples, snapshots, suggestions for 3 components in TK-2**

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## TK-2

- Organize and count with numbers
- Compare and order numbers
- Learn to add and subtract, using numbers flexibly



## CH. 3: Number Sense

<b>California Preschool Learning Foundations Mathematics</b>	<b>California Common Core State Standards for Kindergarten Mathematics</b>
Number Sense	Counting and <b>Cardinality</b>
Children understand numbers and quantities in their everyday environment.	<ul style="list-style-type: none"><li>• Know number names and the count sequence.</li><li>• Count to tell the number of objects.</li><li>• Compare numbers.</li></ul>
Children understand number relationships and operations in their everyday environment	Operations and Algebraic Thinking <ul style="list-style-type: none"><li>• Understand addition as putting together and adding to, and subtraction as taking apart and taking from</li></ul> Number and Operations in Base Ten <ul style="list-style-type: none"><li>• Work with numbers 11–19 to gain foundations for place value</li></ul>

# CH. 6: Mathematics Investigating and Connecting TK-5

Figure 6.3 Progression of Big Ideas, Transitional Kindergarten Through Grade Two

Content Connections	Big Ideas: Transitional Kindergarten	Big Ideas: Kindergarten	Big Ideas: Grade One	Big Ideas: Grade Two
Reasoning with Data	Measure and Order	Sort and Describe Data	Make sense of Data	Represent Data
Reasoning with Data	Look for Patterns	n/a	Measuring with Objects	Measure and Compare Objects
Exploring Changing Quantities	Measure and Order	How Many?	Measuring with Objects	Dollars and cents
Exploring Changing Quantities	Count to 10	Bigger or Equal	Clocks and Time	Problem solving with measures
Exploring Changing Quantities	n/a	n/a	Equal Expressions	n/a
Exploring Changing Quantities	n/a	n/a	Reasoning about Equality	n/a
Taking Wholes Apart, Putting Parts Together	Create Patterns	Being flexible within 10	Tens and Ones	Skip Counting to 100
Taking Wholes Apart, Putting Parts Together	Look for Patterns	Place and position of numbers	n/a	Number Strategies
Taking Wholes Apart, Putting Parts Together	See and use Shapes	Model with numbers	n/a	n/a
Discovering shape and space	See and use shapes	Shapes in the world	Equal parts inside shapes	Seeing fractions in shapes
Discovering shape and space	Make and measure shapes	Making shapes from parts	n/a	Squares in an array
Discovering shape and space	Shapes in space	n/a	n/a	n/a

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## CH. 6: Mathematics Investigating and Connecting TK-5

Integrating **CH 1, 2, 3, 4, 5** specifically for **TK-2** and then **3-5** while **illustrating:**

Tasks

Language Routines

UDL

Classroom  
Snapshots &  
Vignettes

Supporting  
Developmental  
Trajectories

Productive  
Strategies  
for Diverse  
Learners

Orchestrating

## 2. Provide Entry Points for Use and Implementation



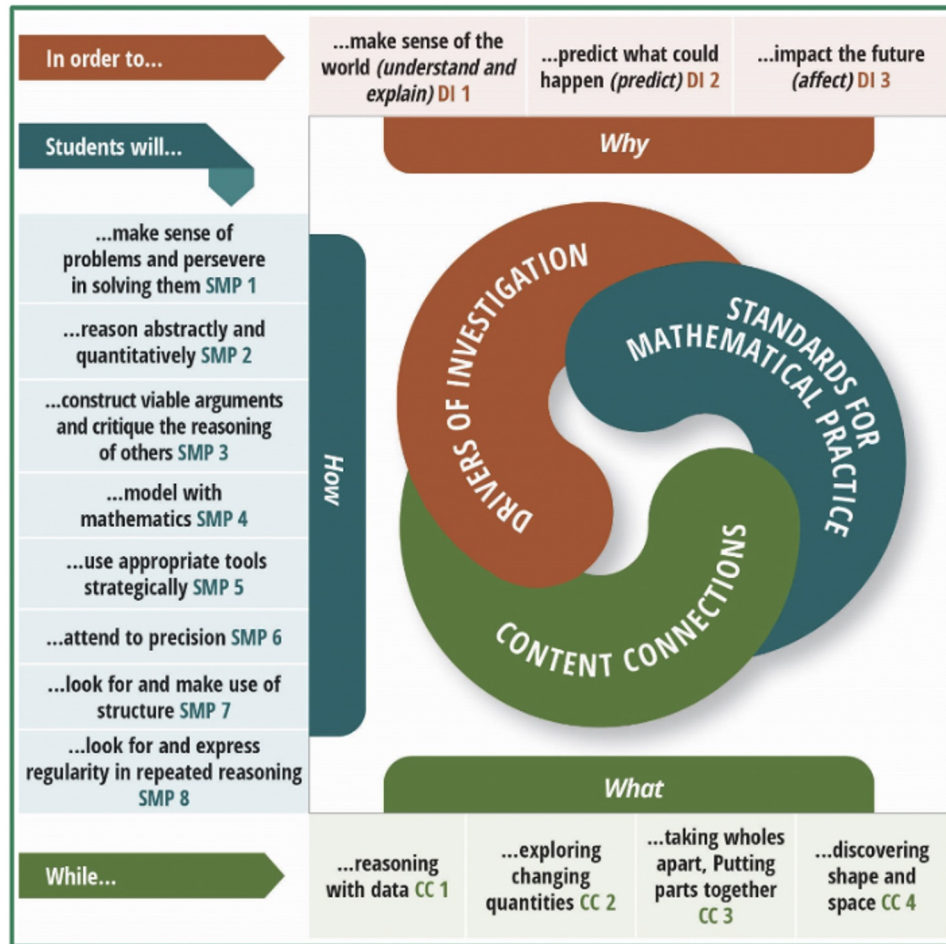
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# ENTRY POINTS

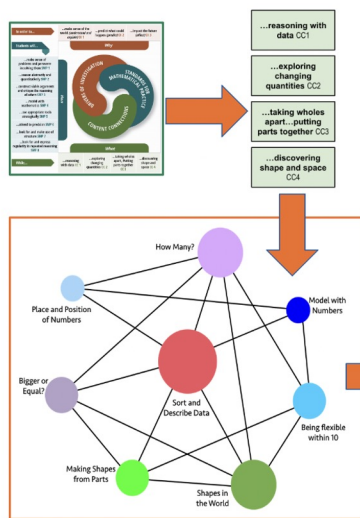
## For Individuals and PLCs

*“How does this approach  
(Why/How?What) align with  
our aspirations for our  
children?”*

*“How might this support how  
we think, talk, and plan?”*



# ENTRY POINTS



Content Connections	Big Ideas	Kindergarten Content Standards
Exploring Changing Quantities	<b>Bigger or Equal?</b>	CC.4, CC.5, CC.6, MD.2, G.4: Identify a number of objects as greater than, less than, or equal to the number of objects in another group. Justify or prove your findings with number sentences and other representations.
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## Engage with the Big Ideas

To make connections from  
Preschool Foundations to  
K Standards

Alternative to “essential  
standards”

# ENTRY POINTS

For Individuals and  
PLCs

Chapter 2 - Take a  
component (e.g., 5)

READ

Discuss

Explore Vignettes  
(Appendix C)

## FIVE COMPONENTS OF EQUITABLE AND ENGAGING TEACHING FOR ALL STUDENTS



COMPONENT 1:  
PLAN TEACHING AROUND BIG IDEAS



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excerpted from the 2023 California Mathematics Framework

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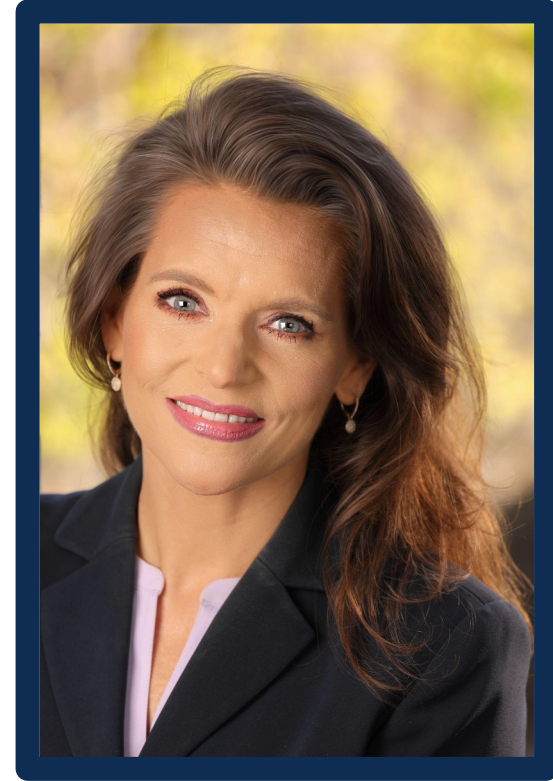
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WHERE ELSE?  
So many  
points of entry  
So much  
support

# THANK YOU!!!!!!



**Ellen Barger**  
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